Effects of a Thermal Bath of Photons on Embedded String Stability

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We compute the corrections of thermal photons on the effective potential for the linear sigma model of QCD. Since we are interested in temperatures lower than the confinement temperature, we consider the scalar fields to be out of equilibrium. Two of the scalar field are uncharged while the other two are charged under the U(1) gauge symmetry of electromagnetism. We find that the induced thermal terms in the effective potential can stabilize the embedded pion string, a string configuration which is unstable in the vacuum. Our results are applicable in a more general context and demonstrate that embedded string configurations arising in a wider class of field theories can be stabilized by thermal effects. Another well-known example of an embedded string which can be stabilized by thermal effects is the electroweak Z-string. We discuss the general criteria for thermal stabilization of embedded defects.

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I. INTRODUCTION

Topological defects can play an important role in early universe cosmology (see e.g. [1–3] for overviews). On one hand, particle physics models which yield defects such as domain walls which have problematic and unobserved effects can be ruled out. On the other hand, topological defects may help explain certain cosmological observations. They could contribute to structure formation or generate primordial magnetic fields which are coherent on cosmological scales [4].

The Standard Model of particle physics does not give rise to topological defects which are stable in the vacuum. On the other hand, it is possible to construct string-like configurations which would be topological defects if certain of the fields were constrained to vanish. If they are not, then the defects are unstable in the vacuum. Such defects are called "embedded defects" (see [5] for an overview). Two prime examples of such embedded defects are the pion string arising in the low energy linear sigma model of QCD [6], and the electroweak Z-string [7] arising in the electroweak theory.

Both the pion string and the electroweak Z-string arise in models with two complex scalar fields, one of them uncharged with respect to the $U(1)$ gauge field of electromagnetism, the second charged. In terms of real fields, we have four real scalar fields $\phi_i, i = 0, .., 3$ with a bare potential of the form

$$V(\phi) \,=\, \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2 \,, \tag{1}$$

where $\phi^2 \,=\, \sum_{i=0}^{3} \phi_i^2$. In both physical examples, two of the fields (ϕ_0 and ϕ_3) are uncharged whereas the two others are charged. Since the vacuum manifold of this theory is $\mathcal{M} = S^3$, there are no stable topological defects, only Π_3 defects which in cosmology are called textures [8].

However, the presence of an external electromagnetic field breaks the symmetry since only two of the fields couple to the photon field. Interactions with the photon field lift the potential in the charged field directions, leading to a reduced symmetry group G which is

$$G \,=\, U(1)_{global} \times U(1)_{gauge} \tag{2}$$

instead of $O(4)$. The vacuum manifold becomes a circle $\mathcal{M} = S^1$ corresponding to

$$\phi_1 = \phi_2 = 0, \quad \phi_0^2 + \phi_3^2 = \eta^2 \,. \tag{3}$$

It is possible to construct embedded cosmic string solutions which are topological cosmic strings of the reduced theory with $\phi_1 = \phi_2 = 0$. The field configuration of such a string (centered at the origin of planar coordinates and extended along the z-axis) takes the form

$$\Phi(\rho, \theta) \,=\, f(\rho)\eta e^{i\theta} \,, \tag{4}$$

where the complex electrically neutral field is $\Phi = \phi_0 + i\phi_3$. $f(\rho)$ is a function which interpolates between $f(0) = 0$ and $f(\rho) = 1$ for $\rho \to \infty$ with a width which is of the order $\lambda^{-1/2}\eta^{-1}$. In the above, ρ and θ are the polar coordinates in the plane perpendicular to the z-axis.

In [9], a plasma stabilization mechanism for the pion string and the electroweak Z-string was proposed. The argument was based on interpreting the terms in the covariant derivative which couple the charged scalar field $\pi^+ = (1/\sqrt{2})(\phi_1 + i\phi_2)$ to the gauge field as a term which, if the gauge field is in thermal equilibrium, will add a term proportional to $\delta V \sim e^2 T^2 |\pi^+|^2$, to the effective potential of the scalar field sector. This lifts the potential in the direction of the charged scalar fields, leaving us with a reduced vacuum manifold given by (3).

In this paper we put this suggested stabilization mechanism on a firmer foundation, focusing on the example of the pion string. The setting of our analysis is the following: we are interested in temperatures below the chiral symmetry breaking transition. Hence, the scalar fields are out of thermal equilibrium. However, the photon field is in thermal equilibrium. This is not the usual setting for finite temperature quantum field theory (since not all fields are in thermal equilibrium) and hence non-standard techniques are required.

We compute the effective potential for the scalar fields obtained by integrating out the gauge field, taking it to be in thermal equilibrium. To do so we use a functional integral in which the time domain is Euclidean and ranges from 0 to β, where β is the inverse temperature. We find that the resulting scalar field effective potential has a broken symmetry and a vacuum manifold given by (3). There is hence an energetic barrier which has to be overcome to destroy a pion string.

II. THE PION STRING

The cosmological context of this work is Standard Big Bang Cosmology. At about 10 microseconds after the Big Bang a phase transition from the quark-gluon plasma to a hadron gas is expected to have taken place at a temperature of about $T_{QCD} \sim 150-200$MeV. Below this critical temperature, the physics of hadrons can be well described by a linear sigma model of four scalar fields which we collectively denote by ϕ, three of them representing the pions. If we make the standard assumption that the relevant bare quark masses vanish, i.e. $m_u = m_d = 0$, where the subscripts stand for the up and down quark, respectively, we know that the effective action (in vacuum) will have a $SU(2)$ symmetry which is spontaneously broken by a potential $V(\phi)$. This spontaneous symmetry breaking leads to a mass for the fourth scalar field, the so-called sigma field.

In the setup described above, the "vacuum manifold" $\mathcal{M}$, i.e. the set of field configurations which minimize the potential, is nontrivial and takes the form of a 3-sphere S^3. There are no stable topological defects associated with this symmetry breaking, only global textures which are not stable. In particular, since the first homotopy group of the vacuum manifold is trivial, i.e. $\Pi_1(\mathcal{M}) = 1$, there are no stable cosmic strings.

However, as discussed in [6], it is possible to construct embedded strings for which the charged pion fields are set to zero, and the two neutral fields (the neutral pion and the sigma fields) form a cosmic string configuration. This is the so-called "pion string".

Since two of the four scalar fields are charged (the two charged pion fields) while the remaining two are neutral, turning on the electromagnetic field will destroy the $O(4)$ symmetry and will break it down to $U(1)_{global} \times U(1)_{local}$. The second factor corresponds to rotations of the charged complex scalar field, the first to a rotation of the neutral one. The pion string can be viewed as the cosmic string configuration associated with the first $U(1)$.

As a toy model for the analytical study of the stabilization of embedded defects by plasma effects we consider the chiral limit of the QCD linear sigma model, involving the sigma field σ and the pion triplet $\vec{\pi} = (\pi^0, \pi^1, \pi^2)$, given by the Lagrangian

$$\mathcal{L}_0 \,=\, \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \eta^2)^2 \,, \quad (5)$$

where η^2 is the ground state expectation value of $\sigma^2 + \vec{\pi}^2$. In the following, we denote the potential in (5) by V_0.

Two of the scalar fields, the σ and π_0, are electrically neutral, the other two are charged. Introducing the coupling to electromagnetism, it is convenient to write the bosonic sector $\mathcal{L}$ of the resulting Lagrangian in terms of the complex scalar fields

$$\pi^+ \,=\, \frac{1}{\sqrt{2}}(\pi^1 + i\pi^2),\ \pi^- \,=\, \frac{1}{\sqrt{2}}(\pi^1 - i\pi^2)\,. \quad (6)$$

According to the minimal coupling prescription we obtain

$$\mathcal{L} \,=\, \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 + D^+_\mu\pi^+D^{\mu-}\pi^- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + V_0\,,$$

where $D^+_\mu \,=\, \partial_\mu + ieA_\mu$, $D^-_\mu \,=\, \partial_\mu - ieA_\mu$.

Effective pion-photon interactions appear through the covariant derivative. They break the $O(4)$ symmetry which the Lagrangian would have in the absence of the gauge field.

In the following we work in terms of the two complex scalar fields

$$\begin{aligned} \Phi \,&=\, \sigma + i\pi_0 \\ \pi_c \,&=\, \pi_1 + i\pi_2\,, \end{aligned} \quad (7)$$

the first of which is electrically neutral, the second charged.

The minimum of the potential can be obtained for $\langle\pi_c\rangle = 0$ and $\langle\Phi\rangle = \eta$ and electromagnetism is unbroken. In that case, the vacuum manifold S^3 reduces to S^1 and some string configurations exist. They are not topologically stable since they can unwind by exciting the charged fields, i.e. $\langle\pi_c\rangle \neq 0$. This string solution is the "pion string". The field Φ vanishes in the center of the string (the charged field vanishes everywhere) and this implies that there is trapped potential energy along the string.

Note that if we distort the field configuration to have $\langle\pi_c\rangle \neq 0$, then the U(1) of electromagnetism gets broken and there is a magnetic flux of $\frac{2\pi}{e}$ in the core of the string. To see how this flux may arise, consider as starting point the pion string configuration with π_c vanishing everywhere. To see the effect of the charged field, let us now consider exciting a constant π_c in the core of the string and see how the potential energy evolves in this case. We easily find that as long as $|\pi_c| \ll \eta$, we get an increase in the potential energy in the core of the string:

$$V(\phi = 0, \pi_c) \,\simeq\, V_0(0,0) + \sqrt{\lambda V_0}|\pi_c|^2 \quad (8)$$

and the lowest potential energy configuration is obtained for $\pi_c = 0$. Based on potential energy arguments alone we would infer that we could get a stable string. However, the kinetic and gradient energies lead to an instability of the vacuum pion string configuration.

We want to consider the effect of a photon plasma on the stability of the pion string. Here we consider an ultra-relativistic plasma of photons. The effective Lagrangian

that describes such a plasma is [10] :

$$\mathcal{L}_{eff} \;=\; \mathcal{L}_{QED} \;+\; \mathcal{L}_\gamma \tag{9}$$

We do not consider the electrons in the Lagrangian for QED, but simply take

$$\mathcal{L}_{QED} \;=\; -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,, \tag{10}$$

where $F_{\mu\nu}$ is the field strength associated with the electomagnetic 4-potential A_μ.

The plasma terms in the effective Lagrangian (9) are

$$\mathcal{L}_\gamma \;=\; \frac{3}{4}\, m_\gamma^2 \, F_{\mu\alpha} \left\langle \frac{K^\alpha K^\beta}{(K\cdot\partial)^2} \right\rangle F^\mu{}_\beta \,, \tag{11}$$

where K^α is the four-vector which represents the momentum of the hard field in the loop, and m_γ is the thermal photon mass which is given by

$$m_\gamma^2 \;=\; \frac{e^2}{9}\left(T^2 + \frac{3}{\pi^2}\mu^2\right) \,, \tag{12}$$

and μ is the quark chemical potential.

What really matters is not the plasma behaviour but rather its influence on a charged scalar field. We will use the “Hard Thermal Loop” formalism, that is we work in the limit where the plasma temperature is much higher than any momentum or mass scale in the problem. In our case we have the following for a scalar field ϕ in the fundamental representation of the gauge group [11]:

$$\mathcal{L}_s \;=\; \frac{3}{4}\, m_s^2 \, \phi^\dagger \left\langle \frac{D^2}{(K\cdot D)^2} \right\rangle \phi \tag{13}$$

This reduces to

$$\mathcal{L}_s = m_s^2 \phi^\dagger \phi \tag{14}$$

where K is the moment of the hard field in the loop and $m_s = \frac{eT}{2}$ is the thermal scalar mass (see [12] for details).

Applying the above to our charged field π_c, the effective Lagrangian becomes

$$\begin{aligned}\mathcal{L} \;&=\; -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0 \\ &\quad + D_\mu^+\pi^+ D^{\mu -}\pi^- - V_0 - \frac{e^2T^2}{4}\pi^+\pi^- \,. \end{aligned} \tag{15}$$

This gives rise to a new effective potential while retaining the gauge invariance of the Lagrangian:

$$V_{eff} \;=\; \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - \eta^2)^2 + \frac{e^2T^2}{4}\pi^+\pi_- \,, \tag{16}$$

where $\frac{e^2T^2}{4}\pi^+\pi^- \;=\; \frac{e^2T^2}{8}(\pi_1^2+\pi_2^2) = \frac{e^2T^2}{8}|\pi_c|^2$.

Based on potential energy considerations, the induced terms in the effective potential should lead to the stabilization of pion strings.

III. EFFECTIVE POTENTIAL COMPUTATION

An improved way to study the stability of the pion string in the presence of a thermal bath is to determine the effective potential of the scalar fields (which are out of thermal equilibrium) in the presence of a thermal bath of photons. This effective potential can be obtained by computing the finite temperature functional integral over the gauge field, treating the scalar fields as external out-of-equilibrium classical ones. They are out of thermal equilibrium since their masses are heavy compared to the temperature if we are below the critical temperature. Note also that string configurations are out-of-equilibrium states below the Ginsburg temperature (which is the temperature slightly lower than the critical temperature when the defect network freezes out during the symmetry breaking phase transition [1–3]).

We make use of the imaginary time formalism [13] of thermal field theory (see e.g. [14] for a review). We work in Euclidean space-time: $t \to i\tau$ and $\tau : 0 \to T^{-1} = \beta$.

The starting point is the action

$$\begin{aligned} S[A^\mu,\Phi,\pi_c] &= \int d^4x[\tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \tfrac{\lambda}{4}(|\Phi|^2+|\pi_c|^2-\eta^2)^2] \\ &\quad + \int_0^\beta d\tau \int d^3x[D_\mu^+\pi^+D^{\mu-}\pi^- - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}] \end{aligned} \tag{17}$$

where the space integral runs from $-\infty$ to $+\infty$.

In the imaginary time formalism of thermal field theory, the integration over four-momenta is carried out in Euclidean space with $k_0 = ik_4$, this means that the transition from zero temperature field theory is obtained via $\int \frac{d^4k}{(2\pi)^4} \to i\int \frac{d^4k_E}{(2\pi)^4}$. Next, we recall that boson energies take discrete values, namely $k_4 = \omega_n = 2n\pi T$ with n an integer, and thus

$$\int \frac{d^4k_E}{(2\pi)^4} \to T\sum_n \int \frac{d^3k}{(2\pi)^3}. \tag{18}$$

We use this Matsubara mode decomposition for the gauge field only, because it is the only field in thermal equilibrium.

The standard definition for the effective potential is based on the Legendre transform of the generating functional (see [14, 15] for reviews). However, the finite temperature effective potential with the scalar fields viewed as classical background fields can also be defined as

$$\begin{aligned} Z[T] \;&=\; \int \mathcal{D}\Phi\mathcal{D}\pi_c\mathcal{D}A^\mu e^{-S[A^\mu,\Phi,\pi_c]} \\ &=\; \int \mathcal{D}\Phi\mathcal{D}\pi_c e^{-S[\Phi,\pi_c]} e^{-\frac{V_{eff}(\Phi,\pi_c)V}{T}} \end{aligned} \tag{19}$$

where $S[\Phi,\pi_c]$ is the gauge field independent part of the (non-euclidean) action, V is the volume of the system and $\int d\tau d^3\mathbf{x} = \frac{\mathbf{V}}{\mathbf{T}}$.

To evaluate the total partition function $Z[T]$ of the system we work in the covariant Feynman gauge following the procedure reviewed e.g. in [16], according to which

the two unphysical degrees of freedom of A^μ that correpond to the longitudinal and timelike photons are cancelled by the ghost and anti-ghost fields c and $\bar{c}$:

$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}A_\mu e^{-S[\Phi,\pi_c]} \tag{20}$$
$$\times e^{-\int_0^\beta d\tau\int d^3x\ \bar{c}(-\partial^2 - e^2|\pi_c|^2)c} e^{-\int_0^\beta d\tau\int d^3x\ \frac{1}{2}A_\mu(\partial^2+e^2|\pi_c|^2)A_\mu}$$

Here the summation of $A_\mu A_\mu$ is in Euclidean space since $A_0 \to iA_0$. We can see from above that the gauge field obtains an effective mass equal to $m_{eff} = e|\pi_c|$. Now we simply evaluate the Gaussian integration over the gauge field and the ghost fields.

$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c e^{-S[\Phi,\pi_c]}$$
$$\times e^{2\frac{1}{2}Tr[\ln(\omega_n^2+\mathbf{k}^2+m_{eff}^2)]}e^{-4\frac{1}{2}Tr[\ln(\omega_n^2+\mathbf{k}^2+m_{eff}^2)]}$$
$$Z[T] = \int \mathcal{D}\Phi\mathcal{D}\pi_c e^{-S[\Phi,\pi_c]}e^{-Tr[\ln(\omega_n^2+\mathbf{k}^2+m_{eff}^2)]} \tag{21}$$

Comparing (21) with the definition (19) of the effective potential we find

$$V_{eff}(\Phi,\pi_c,T) = V_0 + \lim_{V\to\infty}\frac{T}{V}\sum_{n\in\mathbb{Z}}\ln(\omega_n^2+\mathbf{k}^2+m_{eff}^2) + \text{cst}$$
$$= \frac{\lambda}{4}(|\Phi|^2+|\pi_c|^2-\eta^2)^2 + 2\int\frac{d^3k}{(2\pi)^3}[\frac{\omega}{2}+T\ln(1-e^{-\frac{\omega}{T}})] \tag{22}$$

where $\omega = \sqrt{\mathbf{k}^2+m_{eff}^2}$ and $V_0 = \frac{\lambda}{4}(|\Phi|^2+|\pi_c|^2-\eta^2)^2$. The thermal part, $J(m_{eff},T) = \int\frac{d^3k}{(2\pi)^3}T\ln(1-e^{-\frac{\omega}{T}})$, admits a high-temperature expansion :

$$\begin{aligned} J(m_{eff},T) &= \frac{T}{2}\sum_n\int\frac{d^3k}{(2\pi)^3}\ln(\omega_n^2+\mathbf{k}^2+m_{eff}^2) \\ &\simeq -\frac{\pi^2T^4}{90}+\frac{m_{eff}^2T^2}{24}-\frac{m_{eff}^3T}{12\pi} \\ &\quad -\frac{m_{eff}^4}{32\pi^2}\left[\ln\left(\frac{m_{eff}e^{\gamma_E}}{4\pi T}\right)-\frac{3}{4}\right]+\mathcal{O}(\frac{m_{eff}^6}{T^2})\,. \end{aligned} \tag{23}$$

The zero temperature part, $J_0(m)$, is UV divergent :

$$\begin{aligned} J_0(m) &= \int\frac{d^3p}{(2\pi)^3}\frac{\omega}{2}\bigg|_{\omega=\sqrt{p^2+m^2}} \\ &= -\frac{m^4\mu^{-2\epsilon}}{64\pi^2}\left[\frac{1}{\epsilon}+\ln\frac{\overline{\mu}^2}{m^2}+\frac{3}{2}+O(\epsilon)\right]. \end{aligned} \tag{24}$$

The renormalized value of this integral has been obtained using the $\overline{MS}$ renormalization parameter $\bar{\mu}$. $d = 3-2\epsilon$ is the dimension of the momentum integral. Considering renormalization of coupling constants as well, will give $O(\hbar)$ corrections to the potential, but should not change the topology of the vacuum manifold. For example, if the vacuum manifold is a circle it could become a titled circle but since these are small effects, this will not affect the presence of a topological defects.

At high-temperatures we can truncate the series in (23) and get

$$\begin{aligned} V_{eff}(\Phi,\pi_c,T) &= \frac{\lambda}{4}(|\Phi|^2+|\pi_c|^2-\eta^2)^2-\frac{\pi^2T^4}{45}+\frac{e^2|\pi_c|^2T^2}{12} \\ &\quad -\frac{e^3|\pi_c|^3T}{6\pi}-\frac{e^4|\pi_c|^4}{16\pi^2}\left[\ln\left(\frac{e|\pi_c|e^{\gamma_E}}{4\pi T}\right)-\frac{3}{4}\right] \end{aligned}$$

where we neglect terms of order $\frac{e^6|\pi_c|^6}{T^2}$. This potential is also the approximate potential close to the (0,0) point of the (Φ,π_c)-plane. For consistency with the hard thermal loop approximation, we can check that, when the gauge field takes on an effective mass, m_{eff}, it has 3 polarizations instead of 2. This leads to the appearance of an overall factor of $\frac{3}{2}$ for the thermal part of the effective potential. The term quadratic in temperature then becomes $m_{eff}^2T^2/8$ as it appears in (16).

The above computation shows that the effects of the photon plasma create an energy barrier which lifts the scalar field effective potential in direction of the charged fields. In order to minize this effective potential, the charged fields must go to zero. The "effective" vacuum manifold $\mathcal{M}$ is now no longer S^3 but rather $\mathcal{M} = S^1$ which has nontrivial first homotopy group and hence admits stable cosmic string solutions which are precisely the pion strings discussed earlier.

IV. CONCLUSIONS

We have studied the effective potential for the scalar fields of the low-energy effective sigma model of QCD in the presence of a thermal bath of photons. We have shown that the plasma effects break the symmetry of the theory, lift the potential in direction of the charged pion fields, and lead to an effective vacuum manifold which admits cosmic string solutions, the pion strings.

Our analysis puts the stabilization mechanism of [9] on a firmer footing. It shows that if pion strings form, they will be stabilized by plasma effects, at least at a classical level. The stability of pion strings to quantum processes remains to be studied.

The analysis in this paper applies not only to pion strings, but equally to the corresponding embedded strings in the electroweak theory, the Z-string. Thus, below the confinement scale the Standard Model of particle physics admits two types of classically stable embedded strings.

Our arguments, however, are more general and apply to many theories beyond the Standard Model. Given a theory with a multi-component scalar field order parameter with one set of components which are neutral, and a second set which are charged - neutral and charged being with respect to the fields excited in the plasma. Then topological defects of the theory with vanishing charged order parameters become embedded defects of the full theory with the property that they are stabilized in the early universe.

Stabilized embedded defects can have many applications in cosmology. For example, stabilized pion strings provide an explanation for the origin and large-scale coherence of cosmological magnetic fields [4]. On the other hand, stabilized embedded domain walls would lead to an overclosure problem. Hence, theories admitting those types of embedded defects would be ruled out by cosmological considerations.

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